

Regular Expressions

CSCI 2670: Introduction to Theory of Computing

Farhan Jiva

Outline

- Introduction
- Vocabulary Recap
- Definitions
 - Constants
 - Operators
- Operator Precedence
- Examples
- Group quiz

Introduction

- A regular expression provides for a concise and flexible means for “matching” strings of text
 - i.e. certain characters, words, patterns of characters
- The concept of regular expressions was first popularized by Unix tools
 - ed, grep
- Invented by Stephen Cole Kleene in the 1950s

Vocabulary Recap

- Recall from our last class:
 - A character is some type of symbol (letter, digit)
 - An alphabet (denoted by Σ) is any set of characters
 - A word (or string) is any finite sequence of characters over an alphabet Σ

Definition - Constants

- Given a finite alphabet Σ
 - Empty set (\emptyset) is a set containing zero elements
 - Empty string (λ) is a string containing zero characters
 - Literal character (a) is a set containing only the character a

Definition - Concatenation Operator

- Simplest type of regular expression
- Formed by concatenating many characters together
- Formal definition
 - RS denoting the set $\{ \alpha\beta \mid \alpha \text{ in } R \text{ and } \beta \text{ in } S \}$
- Examples
 - Alphabet $\Sigma = \{a,b\}$
 - Language = $\Sigma\Sigma$
 - String in the language: aa, ab, ba, bb

Definition - Alteration Operator

- Lets us choose from one of many possibilities
 - Similar to the logical OR operator
- Formal definition
 - $R|S$ denoting the set union of R and S
- Examples
 - Alphabet $\Sigma = \{a,b\}$
 - Language = $aa | bb$
 - Strings in the language: exactly two strings, aa and bb

Definition - Kleene Star Operator

- Also known as the “replication” operator
- Formal definition
 - R^* denoted the set of all strings that can be made by concatenating any finite number (including zero) of strings from R
- Examples
 - Alphabet $\Sigma = \{a,b\}$
 - Language = a^*
 - Strings in the language: $\lambda, a, aa, aaa, aaaa, \dots$

Definition - Plus Operator

- Not a fundamental operator, however used extensively in the real world.
- Similar to the Kleene Star operator, however the preceding character must match at least once
- Examples
 - Alphabet $\Sigma = \{a,b\}$
 - Language = a^+b^+
 - Strings in the language: $ab, aabb, abb, aab, \dots$
 - Note that the empty string is NOT in the language described above
- Plus operator can also be written using the Kleene Star
 - $a^+ \Leftrightarrow aa^*$

Definition - Grouping Operator

- Allows us to specify precedence (with parentheses) to the various operators
- Examples
 - Alphabet $\Sigma = \{a,b\}$
 - Language = $(ab)^*(a|b)$
 - Strings in the language: a, aba, ababb, b, ...

Operator Precedence

- Order of operations for regular expression operators
 - Parentheses
 - Kleene star/plus operator
 - Concatenation
 - Alteration
- If there are no ambiguities, parentheses may be omitted
 - $(ab)c = abc$
 - $a|(b(c^*)) = a|bc^*$

Examples

- Alphabet $\Sigma = \{a,b\}$
- $a = a$
- $a | b = a, b$
- $(a | b)(b | a) = aa, ab, bb, ba$
- $a^*b^* = \lambda, a, aa, aaa, b, bb, bbb, ab, aabb, \dots$
- $(bb)^* = \lambda, bb, bbbb, bbbbbb, \dots$
- $(ab)^+ | a^+ = ab, ababab, a, aa, aaa, \dots$

Group Quiz

- Alphabet $\Sigma = \{a,b\}$
- Group 1
 - $(a+b^+)(ab)^*$
- Group 2
 - $ba^*b^+ | ab^*a^+$
- Group 3
 - $a^*b^* | ba^+$
- Group 4
 - $(b^+(a^+)^*(ab)^+ | (a|b)^*$